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FOUR YEAR HONOURS B.Sc. DEGREE EXAMINATION, JUNE/JULY - 2024

CHOICE BASED CREDIT SYSTEM

SECOND SEMESTER - MAJOR

Part - II : Mathematics

Paper -3 : Differential Equations

(Under CBCS New Regulation w.e.f the academic year 2023-24)

Time : 3 Hours **Max. Marks : 75**

SECTION - A

Answer any Five of the following questions. Each question carries equal marks. **(5×5=25)**

1. Solve $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

2. Solve $(1+xy)ydx+(1-xy)xdy=0.$

3. Solve $y+px=x^4p^2, p=\frac{dy}{dx}$

4. Find the orthogonal trajectories of $r=c(1-\sin\theta), c$ is arbitray constant.

5. Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$ with $y=0, \frac{dy}{dx}=0$ *when* $x=0.$

6. Solve $\left(D^2-2D-3\right)y=5.$

7. Find the particular integral of $(D^2-2D+5)y=e^{2x}\sin x$

8. Find the general solution of $(D^2+4D+4)y=3xe^{-2x}$

9. Find the equivalent differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=x^2$ with the substitution $x=e^t$

10. Find the other independent solution of $x^2y''-xy'+y=0, y_1=x$ is a one given solution.

SECTION - B

Answer All questions. Each question carries equal marks. **(5×10=50)**

11. a) Let M and N are two functions in x and y whose first order partial derivatives are continuous. Prove that the sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(OR)

b) Solve $\left(1-x^2\right)\frac{dy}{dx} + 2xy = x\left(1-x^2\right)^{\frac{1}{2}}, |x| < 1.$

12. a) Solve $p^2 + 2py\cot x = y^2, p = \frac{dy}{dx}$.

(OR)

b) Solve $y = yp^2 + 2px, p = \frac{dy}{dx}$.

13. a) Solve $\left(D^2 + a^2\right)y = \sin ax, D = \frac{d}{dx}$.

(OR)

b) Solve $\left(D^2 - m^2\right)y = \cosh mx, D = \frac{d}{dx}$.

14. a) Solve $\left(D^2 + 4D - 12\right)y = (x-1)e^{2x}, D = \frac{d}{dx}$

(OR)

b) Solve $\left(D^2 - 2D + 5\right)y = e^{2x}\sin x, D = \frac{d}{dx}$

15. a) Explain the procedure of method of Variation of Parameters to solve $a_2\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_0y = Q(x), a2 \neq 0$.

(OR)

b) Solve $x^2D^2y - xDy - 3y = x^2\log x, D = \frac{d}{dx}$

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FOUR YEAR B.Sc. HONOURS DEGREE EXAMINATION, JUNE/JULY -2024

CHOICE BASED CREDIT SYSTEM

SECOND SEMESTER MAJOR

PART - II: MATHEMATICS

PAPER -4 : ANALYTICAL SOLID GEOMETRY

(Under CBCS New Regulation w.e.f the academic year 2023-24)

k. Divya

Time : 3 Hours **Max. Marks : 75**

SECTION-A

Answer any Five of the following questions. Each question carries equal marks.

(5×5=25)

1. Find the equation of the plane through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.

2. Find the bisectors of the acute angle between the planes $2x-y-2z+3=0$. and $3x-2y+6z+8=0$.

3. Find the coordinates of the point of intersection of the line $\frac{x+1}{1}=\frac{y+3}{3}=\frac{z-2}{-2}$ plane $3x+4y+5z=5$.

4. Show that the lines $x+2y-5z+9=0=3x-y+2z-5;\ 2x+3y-z-3=0=4x-5y+z+3$ are coplanar.

5. Find the centre and radii of the sphere $2x^2+2y^2+2z^2-2x+4y+2z+3=0$.

6. Prove that the circles $x^2+y^2+z^2-2x+3y+4z-5=0 \quad 5y+6z+1=0;$ $x^2+y^2+z^2-3x-4y+5z-6=0 \quad x+2y-7z=0;$ lie on the same sphere.

7. Find the equation of the sphere that passes through the circle $x^2+y^2+z^2-2x+3y-4z+6=0, 3x-4y+5z-15=0$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11=0$ orthogonally.

8. Obtain the angle between two spheres whose radii r_1 and r_2 and distance between their centres is d.

9. Find the equation of the cone whose vertex is (α,β,γ) *and base* $ax^2+by^2=1, z=0.$

10. Show that the general equation of the cone which passes through the three axes is $fyz+gzx+hxy=0.$

SECTION - B

Answer any Five questions. Each question carries equal marks. **(5×10=50)**

11. a) A plane meets the coordinate axes A,B,C such that the centroid of the triangle ABC is the point (a,b,c). Show that the equation of the plane is $\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3.\right)$

(OR)

b) Show that $12x^2-2y^2-6z^2-2xy+7yz+6zx=0$ represents pairs of planes and find the angles between them.

12. a) Find the shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}, \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}.$

(OR)

b) Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}\frac{x-1}{2}, \frac{y+1}{-3}=\frac{z+10}{8}=$ intersect and find the coordinates of the point of intersection.

13. a) Obtain the equation of the sphere which passes through the three points (1,0,0,), (0,1,0),(0,0,1) and has its radius as small as possible.

(OR)

b) If the tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a,b,c on the coordinate axes, then show that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$

14. a) Find the limiting points of the coaxal system defined by the spheres $x^2+y^2+z^2+3x-3y+6=0; x^2+y^2+z^2-6y-6z+6=0.$

(OR)

b) Find the equations to the two spheres of the coaxal system $x^2+y^2+z^2-5+k(2x+y+3z-3)=0$ which touch the plane $3x+4y=15$.

15. a) Find the vertex of the cone $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$

(OR)

b) Find the equation of the right circular cone whose vertex is (1,-2,-1), axis the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z+1}{5}$ and semi-vertical angle 60^0.